Maths Masterclass, 2005 **Chris Budd, University of Bath**

SYMMETRY AND MATHEMATICS IN FOLK DANCING (1)

Here are some activities to work alone on or to do in groups. Choose any problems that you find interesting and have a go at them.

1. Symmetry can be found all around us. Think of as many possible examples as you can of objects that have symmetry(which can be symmetry under reflection, rotation, or translation).

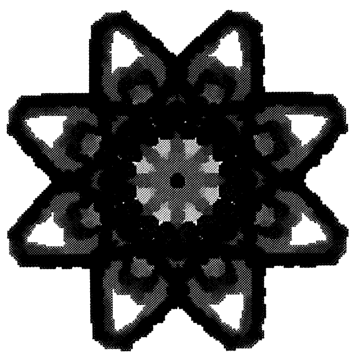
2. Write down all the symmetries of the square and convince yourself that there are four rotations *e, a, aa, aaa* and four reflections, the three that we have already met, *b, c, d* and a new one *f*.

3. Show that $ab = d$, $ba = c$, $bc = a$, and $dc = aa$.

4. See if you can work out how the other operations combine. For example, what are *ac, ad, bd,* etc? If you have time you can work out the 64 different ways of combining two possible operations and fill in the table provided. This is called the *group table*. Without necessarily doing this, can you work out some general rules, for example, what do you get if you combine two reflections, a reflection and a rotation, or two rotations?

5. See if you can repeat questions 2, 3 and 4 for either an equilateral triangle or (for the ambitious) a regular hexagon.

6. By bisecting each side, divide an equilateral triangle into six equal sections. Draw a random pattern in one of these sections. Now, using tracing paper if necessary, reflect this pattern in each of the bisectors and continue to reflect what you have until you have drawn a pattern in the whole triangle. This will give you a pattern based on symmetry that will always look good. This is the basic principle behind a kaleidoscope.



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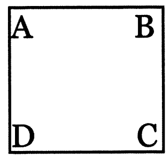
SYMMETRY AND MATHEMATICS IN FOLK DANCING (2)

1. See if you can use the symmetries of a triangle to help you design a dance for three people.

2. Using the symmetry rules for the square devise some new dances.

3. Another way of labelling a square to correspond to the sequence ABCD is as:
Using this new square, work out the effect of the reflection *d* as a rearrangement of the sequence *ABCD*. What is the difference in rearrangement corresponding to the reflection *b* and the rotation *a* from the original square ? Using this new square write out the sequence of moves corresponding to the reel *bcbcbcbc*.



4. Create your own *knitting patterns*.
We have generated several sequences of letters, both in the class and in the above question. Give each letter a different colour and use these to create some attractive "knitting" patterns. For example, see what patterns you get for the simple operation of the repeated rotation *aaaaaaa* in the two cases of the corners of the square arranged either as in the class or in question 2. (Hint: the second pattern could be called stripes.)
A good way to do this is to use some sort of paper with a grid (such as graph paper) and to colour in the squares of the grid corresponding to the sequences of the letters. Experiment boldly!

5. In the American "Grand Square" dance there are eight dancers who stand in an octagon. If we label them clockwise round the octagon they are: *AACDEFGH*. There are three basic moves *a, b, c* such that

$$a: ABCDEFGH \rightarrow HCBEDGFA, \qquad b: ABCDEFGH \rightarrow EFCDABGH$$
$$c: ABCDEFGH \rightarrow ABGHEFCD.$$

a) Draw the octagon and see what the effect of each of these moves is. Are any a reflection or rotation?
b) Show that $aa = bb = cc = e$.
c) A dance sequence is *abc*. How many times do the dancers have to repeat this sequence to get back to their original positions?

6. The dance "Nottingham Swing" can be done with six dancers *ABCDEF* (or indeed six couples, in which each letter represents a couple). There are two basic moves which are repeated to give the dance (together with some swings which do not change the order of the dancers). These moves are very similar to the inner and outer twiddle and are given by
$ABCDEF \rightarrow BADCFE$ and $ABCDEF \rightarrow ACBDEF$
Find a way of labelling a *hexagon* so that these two moves correspond to reflections of the hexagon, and that the combination of the two moves corresponds to a rotation of the hexagon. How many times do you have to combine the moves to get back to the starting position?

7. Use the effect of rotations and reflections of the hexagon to devise some other dances for six dancers.

8. This is a real problem in Morris dancing posed to me by a member of a Morris group. This group has twelve members and they want to design waistcoats for the group, which are to be a 12 x 12 grid of squares made up of patches of twelve different colours. Can you design a waistcoat for them using these colours? The rules are that the waistcoats should have each colour once, and only once, in each row and column. The solution to this problem uses Latin squares and you can read about these in *Mathematical Recreations and Essays* by Rouse Ball and Coexeter.

SYMMETRY AND MATHEMATICS IN FOLK DANCING

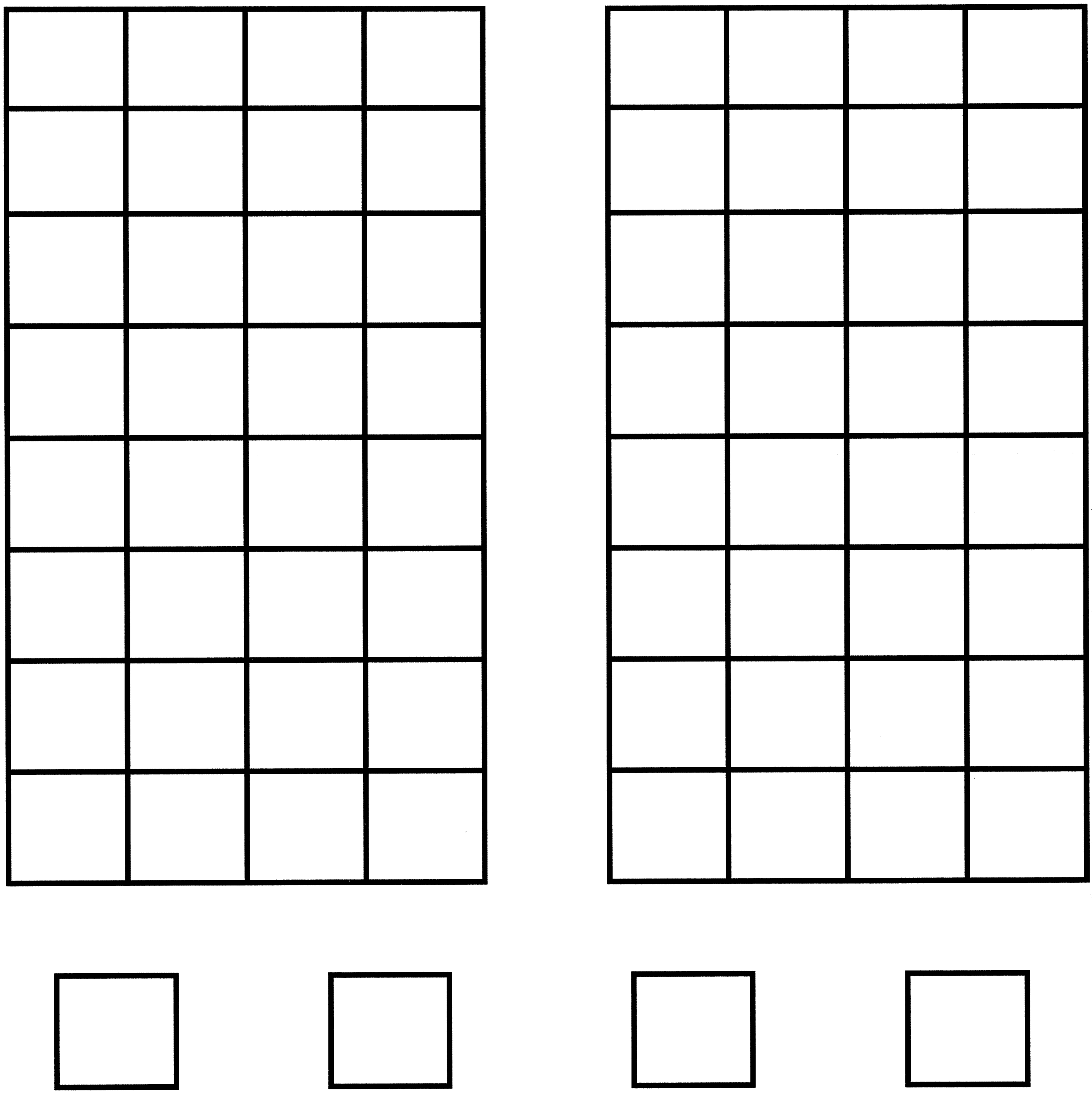
Group Table

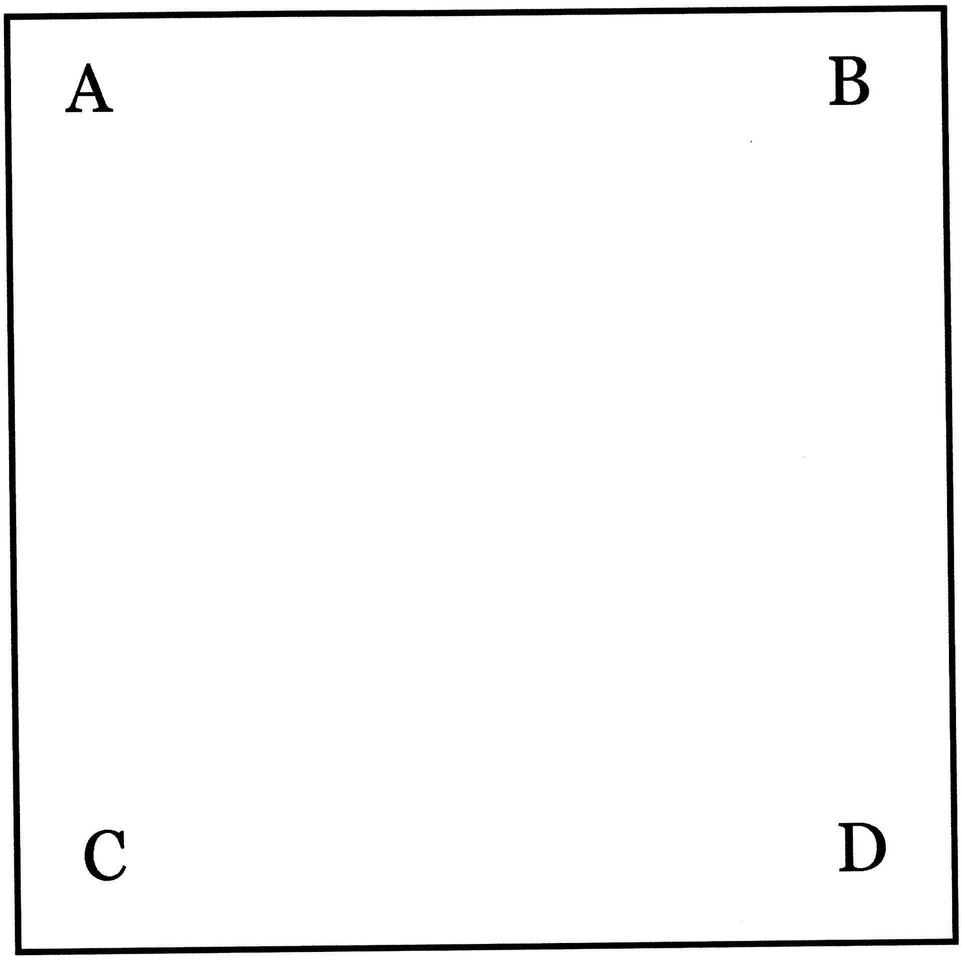
Second

First

	e	a	aa	aaa	b	c	d	f
e		a						
a		aa		e	d			
aa								
aaa								
b						a		
c								
d						aa		
f								

Fill this in by using your square. To help I've put in a few entries.




A
B
C
D